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Pryce-Hoyle Tensor in a Combined Einstein-Cartan-Brans-Dicke Model

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Abstract In addition to introducing matter injection through a scalar field determined by Pryce-Hoyle tensor, we also combine it with a BCDE (Brans-Dicke-Einstein-Cartan) theory with lambda-term developed earlier by Berman (Astrophys. Space Sci. 314:79–82, 2008), for inflationary scenario. It involves a variable cosmological constant, which decreases with time, jointly with energy density, cosmic pressure, shear, vorticity, and Hubble's parameter, while the scale factor, total spin and scalar field increase exponentially. The post-inflationary fluid resembles a perfect one, though total spin grows, but not the angular speed (Berman, in Astrophys. Space Sci. 312:275, 2007). The Pryce-Hoyle tensor, which can measured by the number of injected particles per unit proper volume and time, as well as shear and vorticity, can be neglected in the aftermath of inflation ("no-hair").

Keywords Cosmology · Einstein · Brans-Dicke · Cosmological term · Shear · Spin · Vorticity · Inflation · Einstein-Cartan · Torsion

1 Introduction

The purpose of this paper, is to show that, when exponential inflation is turned on, in the Universe, eventual shear, vorticity, or matter injection (originated from a Pryce-Hoyle term), which may have originated in the very early Universe, are completely erased ("no-hair") by the enormous expansion which represents this phase. We had arrived to this conclusion, when the Pryce-Hoyle tensor is absent, in two previous papers [9, 11]. We shall find the same results, for the present case.

If the Universe is rotating, i.e., if it has a non-zero total-spin, the left-handed creation characteristic [1] of the Universe would be explained; this would also attach a meaning to parity violation, and thus, according to the teachings of [25], it would explain the matter-antimatter asymmetry, plus the Pioneer anomaly [10], and neutrinos left handed spin.

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Berman [14, 15], has shown that Robertson-Walker's metric includes a "hidden" state of rotation plus expansion.

Since the advent of Modern Cosmology [34, 35], two different kinds of models turned-out of the cosmologists' brains: the big-bang, and the stationary. Now that we believe about inflation, as a possible phase of the early Universe, it is difficult to detach the exponential inflationary big-bang model, from the exponential stationary one (for inflation, see [28, 35]). For instance, the reader may check the books by Narlikar for a description of both kinds of models [29, 30]. While matter injection has been put forward with the stationary model, it seems feasible that one could think of such subject, by introducing, into the energy-momentum-tensor employed in the big-bang picture, a Pryce-Hoyle component. This matter injection tensor leads to the so-called C-field, and when we apply to Robertson-Walker's metric, the expanding Universe obeys field equations tantamount to adding the term $\frac{1}{2}\kappa f \dot{\lambda}^2$ to both the energy density and cosmic pressure equations.

Dirac proposed that a time-varying gravitational "constant", could be needed in order to explain some "coincidences" of a cosmological nature [24]. Later, Brans and Dicke [21] included this variation, as a scalar field, in order to accommodate Machian ideas [7, 8, 12]. Scalar fields in 5-D gravitational theories reduce to 4-D theories with a cosmological constant, but the idea of a universal scalar field is present in modern gravitational "scalar-tensor" theories [8]. String theories also introduce "dilaton fields", which are also present in gravitation counterparts. Berman [13] has even calculated energy and momenta of rotating dilaton black holes, which were depicted as possible astrophysical objects.

We shall first review Pryce-Hoyle theory (Sect. 2), then we shall deal (Sect. 3) with the combined torsion plus scalar-tensor gravitational theory, as presented in our recent paper [11], and afterwards, we derive the cosmological solution for a lambda-Universe in an inflationary scenario with matter injection, where the fluid is endowed with shear and vorticity (Sect. 4). In the final Sect. 5, we comment the solution just derived.

2 Review of Pryce-Hoyle Theory

When steady-state theory was devised [30], the stationary exponential Universe led to creation of matter: consider a proper three-volume,

$$V \propto e^{3Ht}$$

Then, we obtain,

$$\frac{\dot{V}}{V} = 3H$$

Consider the constant energy density $\rho = \rho_0$. The amount of matter within the volume V would grow like,

$$\dot{M} = 3HV\rho$$

so that, the rate of creation of matter per unit volume would be something like,

$$Q = 3H\rho$$

As Berman [12] has reported, Hoyle [26], inspired by the above calculation, introduced, in Cosmology, this additional term towards the energy momentum tensor, originated by a scalar field, responsible for matter injection. This field, due to Pryce, Hoyle and Narlikar [17, 27, 30], is represented by:

$$T^{\mu\nu} = T_M^{\mu\nu} - f\left(\lambda^\mu\lambda^\nu - \frac{1}{2}g^{\mu\nu}\lambda^\alpha\lambda_\alpha\right) \tag{1}$$

where, $T_M^{\mu\nu}$ stands for the normal matter energy-momentum tensor, and f is a constant, while λ_μ is a vector given by:

$$\lambda_\mu = \frac{\partial\lambda}{\partial x^\mu} = (0, 0, 0, \dot{\lambda}) \tag{2}$$

Einstein's equations are kept like:

$$G^{\mu\nu} = -\kappa T^{\mu\nu} \tag{3}$$

There is an additional relation,

$$n = j^\mu_{;\mu} \tag{4}$$

which stands for the number of particles injected per unit of proper 4-volume, the particle current being represented by j^μ. For Robertson-Walker's metric,

$$ds^2 = dt^2 - \frac{R^2(t)}{[1 + (\frac{kr^2}{4})]^2} d\sigma^2 \tag{5}$$

where,

$$d\sigma^2 = dx^2 + dy^2 + dz^2 \tag{6}$$

we find the following field equations:

$$\kappa\rho = 3\left(\frac{\dot{R}}{R}\right)^2 + 3\frac{k}{R^2} + \frac{1}{2}\kappa f\dot{\lambda}^2 \tag{7}$$

and,

$$\kappa p = -2\frac{\ddot{R}}{R} - \left(\frac{\dot{R}}{R}\right)^2 - \frac{k}{R^2} + \frac{1}{2}\kappa f\dot{\lambda}^2 \tag{8}$$

Additionally we have an equation for matter injection proper,

$$\ddot{\lambda} + 3\dot{\lambda}\frac{\dot{R}}{R} = f^{-1}n = f^{-1}j^\mu_{;\mu} \tag{9}$$

where $n(t)$ stands for the number of particles injected per unit of proper volume and proper time.

It was Narlikar, in 1973, that noticed that the C-field, could be either considered as representing continuous matter injection or "explosive" big-bang paraphernalia.

For instance, let us work a simple case.

From the field equations, with,

$$R = R_0 e^{Ht} \quad (R_0 = \text{constant}) \tag{10}$$

$$\ddot{\lambda} = 0$$

$$\dot{\lambda} = \frac{n}{3fH} = \text{constant} \tag{11}$$

$$\kappa\rho = 3H^2 + \kappa\frac{f}{2}\dot{\lambda}^2 = 3H^2 + \frac{\kappa}{18f}H^{-2}n^2 = \text{constant} \tag{12}$$

$$\kappa p = -3H^2 + \frac{\kappa}{18f}H^{-2}n^2 = \text{constant} \tag{13}$$

It is supposed, in the above model, that n is constant!!! Whitrow-Randall's relation would also applicable in this case, so that we can call such model as Machian.

3 Review of the Combined BCDE Theory

Berman [9], examined the time behavior of shear and vorticity in a lambda-Universe, for inflationary models, in a Brans-Dicke framework. The resulting scenario is that exponential inflation smooths the fluid, in order to become a nearly perfect one after the inflationary period. In a subsequent paper [11], a similar inflationary scenario was examined with the inclusion of torsion, *à la* Einstein-Cartan, when a scalar field of Brans-Dicke origin, is included, along with a Cosmological lambda-term. Again, with suitable constants, the model performed adequately, and, while total spin grew, along with scale-factor and scalar field, all other characteristics decreased, and the post-inflationary fluid, resembled a perfect one.

Einstein-Cartan's gravitational theory, though not bringing vacuum solutions different than those in General Relativity theory, has an important rôle, by tying macrophysics, through gravitational and electromagnetic phenomena (i.e., involving constants G and c), with microphysics, though Planck's constant, involving spin originated by torsion. Intrinsic angular momentum was introduced by Cartan as a Classical quantity [22] before it was introduced as a Quantum Theory element, around 1925. Of course, spin is important in the Quantum Theory of particles. However, spin has taken part of Classical Field Theory for a long time, and Cosmological models were treated as early as 1973 [33]. Einstein-Cartan Theory is the simplest Poincaré gauge theory of gravity, in the frame of which, the gravitational field is described by means of curvature and torsion, the sources being energy-momentum and spin tensors. It is important to stress that torsion can be originated by spin but not necessarily vice-versa. We mean that Einstein-Cartan's theory, is not the only possible framework for a theory involving spin. Just look at General Relativity Theory, which may include angular momentum phenomena, even without evoking torsion.

Though it was in the past, supposed that, due to spin, Robertson-Walker's metric might not be representative of Physical reality in a torsioned spacetime, recent papers recalled the approach shown by us in several papers [3, 4], on how anisotropic Bianchi-I models in Einstein-Cartan's theory could be reduced to Robertson-Walker's prototype, by defining overall, deceleration parameters, and scale-factors; we did the same thing, with other papers dealing with anisotropic models in GRT and BD theories (for GRT see [2, 19]; for BD theory see [18]). On the other hand, Berman and Som [20] have shown that, slight deviations from Robertson-Walker's metric, changing it to a Bianchi-I metric, are enough to produce the anisotropic phenomena, like entropy production, or other ones; this is a clue to the possibility of considering overall scale-factors and deceleration parameters, etc, in the Raychaudhuri's equation for Einstein-Cartan's Cosmology, without worrying with any anisotropy, which becomes implicit in the equations of Raychaudhuri's book [31]. The essential modification of General Relativistic Bianchi-I cosmology, when we carry towards Einstein-Cartan's, resides, when field equations are explicited, in that the normal energy momentum tensor components T_1^1, T_2^2 and T_3^3 are subtracted by a term S^2, while T_0^0 is added by S^2. Of course, there appear also non-diagonal S-dependent terms: for instance, T_3^2 and T_2^3 depend linearly with S^{32}. In our treatment of the Einstein-Cartan-Brans-Dicke theory, the field equations are obviously satisfied, but we have short-cutted the derivations, like we have done in the previous paper [9], which also conforms with the field equations of that case (Brans-Dicke theory with lambda). The off-diagonal energy momentum components are null, for a Robertson-Walker's framework.

It is generally accepted that scalar tensor cosmologies play a central rôle in the present view of the very early Universe [7]. The cosmological "constant", which represents quintessence, may be a time varying entity, whose origin remounts to Quantum theory [8], but see also a possible Classical explanation for lambda in [16]. The first, and most important scalar tensor theory was devised by Brans and Dicke [21], which is given in the "Jordan's frame". Afterwards, Dicke [23] presented a new version of the theory, in the "Einstein's frame", where the field equations resembled Einstein's equations, but time, length, and inverse mass, were scaled by a factor $\phi^{-\frac{1}{2}}$ where ϕ stands for the scalar field. Then, the energy momentum tensor T_{ij} is augmented by a new term Λ_{ij}, so that:

$$G_{ij} = -8\pi G\left(T_{ij} + \Lambda_{ij}\right) \tag{14}$$

where G_{ij} stands for Einstein's tensor. The new energy tensor quantity, is given by:

$$\Lambda_{ij} = \frac{2\omega + 3}{16\pi G\phi^2}\left[\phi_i\phi_j - \frac{1}{2}G_{ij}\phi_k\phi^k\right] \tag{15}$$

In the above, ω is the coupling constant. The other equation is:

$$\Box \log\phi = \frac{8\pi G}{2\omega + 3}T \tag{16}$$

where $\Box$ is the generalized d'Alembertian, and $T = T_i^i$. It is useful to remember that the energy tensor masses are also scaled by $\phi^{-\frac{1}{2}}$.

For the Robertson-Walker's flat metric,

$$ds^2 = dt^2 - \frac{R^2(t)}{[1 + (\frac{kr^2}{4})]^2}d\sigma^2 \tag{17}$$

$$\text{where } k = 0 \quad \text{and} \quad d\sigma^2 = dx^2 + dy^2 + dz^2$$

The field equations now read, in the alternative Brans-Dicke reformulation [31]:

$$\frac{8\pi G}{3}\left(\rho + \frac{\Lambda}{\kappa} + \rho_\lambda\right) = H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 \tag{18}$$

$$-8\pi G\left(p - \frac{\Lambda}{\kappa} + \rho_\lambda\right) = H^2 + \frac{2\ddot{R}}{R} \tag{19}$$

In the above, we have:

$$\rho_\lambda = \frac{2\omega + 3}{32\pi G}\left(\frac{\dot{\phi}}{\phi}\right)^2 = \rho_{\lambda 0}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{20}$$

From the above equations (18), (19) and (20) we obtain:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho + 3p + 4\rho_\lambda - \frac{\Lambda}{4\pi G}\right) \tag{21}$$

Relation (21) represents Raychaudhuri's equation for a perfect fluid. By the usual procedure, we would find the Raychaudhuri's equation in the general case, involving shear (σ_{ij}) and vorticity (ϖ_{ij}); the acceleration of the fluid is null for the present case, and then we find:

$$3\dot{H} + 3H^2 = 2\left(\varpi^2 - \sigma^2\right) - 4\pi G\left(\rho + 3p + 4\rho_\lambda\right) + \Lambda \tag{22}$$

where Λ stands for a cosmological "constant". As we are mimicking Einstein's field equations, Λ in (22) stands like it were a constant (see however [5–8]). Notice that, when we impose that the fluid is not accelerating, this means that the quadri-velocity is tangent to the geodesics, i.e., the only interaction is gravitational.

When Raychaudhuri's equation is calculated for non-accelerated fluid, taken care of Einstein-Cartan's theory, combined with Brans-Dicke theory, the following equation was found by us, based on the original calculation for Einstein-Cartan's theory by Raychaudhuri [31]:

$$3\dot{H} + 3H^2 = 2\varpi^2 - 2\sigma^2 - 4\pi G\left(\rho + 3p + 4\rho_\lambda\right) + \Lambda + 128\pi^2 S^2 \tag{23}$$

where S stands for the spin density contents of the fluid, where we have omitted a term like

$$\varpi S = \varpi_{ik} S^{ik} + \varpi^{ik} S_{ik} \tag{24}$$

which is to be included in the pressure and energy density terms, by a re-scaling.

The introduction of Pryce-Hoyle tensor, as far as pressure and energy-density are concerned, can be done by the addition of the term $\frac{1}{2}\kappa f\dot{\lambda}^2$, as exposed above. The Raychaudhuri's equation would have the following form for a non-accelerating fluid:

$$3\dot{H} + 3H^2 = 2\varpi^2 - 2\sigma^2 - \frac{\kappa}{2}\left(\rho + 3p + 4\rho_\lambda + \kappa f\dot{\lambda}^2\right) + 128\pi^2 S^2 + \Lambda \tag{25}$$

For inflation, we shall impose, that:

$$3H^2 = \Lambda \tag{26}$$

It is important to stress, that relation (23) is the same general relativistic equation, with the additional spin term, which transforms it into Einstein-Cartan's equation. When we work a combined Einstein-Cartan's and Brans-Dicke theory (BCDE theory), we would need to calculate the new field equations for the combined theory.

By employing the total action [32],

$$L = \int d^4x\sqrt{-g}\left[\pounds_m\left(\psi, \nabla\psi, g\right) - \frac{1}{2\chi}R\left(g, \partial g, Q\right)\right] \tag{27}$$

where the matter Lagrangian contains torsion because the connection is not symmetric, and χ is the coupling constant, both for curvature and torsion, and when we perform independent variations with respect to ψ, $g_{\mu\nu}$ and $Q^\alpha_{\mu\nu}$; the last the one is the torsion tensor,

$$Q^\mu_{\alpha\beta} = \frac{1}{2}\left(\Gamma^\mu_{\alpha\beta} - \Gamma^\mu_{\beta\alpha}\right) \tag{28}$$

We find, the Einstein tensor,

$$G^{\mu\nu} - \hat{\nabla}_\alpha (T^{\mu\nu\alpha} - T^{\nu\alpha\mu} + T^{\alpha\mu\nu}) = \chi T^{\mu\nu} \tag{29}$$

where,

$$T^{\mu\nu\alpha} = \chi S^{\mu\nu\alpha} \tag{30}$$

We have defined,

$$\hat{\nabla}_\alpha \equiv \nabla_\alpha + 2Q_\alpha = \nabla_\alpha + 2Q^{\nu}_{\alpha\nu} \tag{31}$$

while the modified torsion tensor,

$$T^{\alpha}_{\mu\nu} = Q^{\alpha}_{\mu\nu} + \delta^{\alpha}_{\mu} Q_\nu - \delta^{\alpha}_{\nu} Q_\mu \tag{32}$$

4 Solution for BCDE Theory with Matter Injection

From the prior sections, we now are able to write the resulting equations for a perfect fluid, which can be inferred from [31]:

$$-8\pi\left[\frac{1}{2}\kappa f\dot{\lambda}^2 + p\right] = [\text{Brans-Dicke alternative Riemann tensor } G^i_i] + 256\pi^2 S^2 \tag{33}$$

$$8\pi\left[\frac{1}{2}\kappa f\dot{\lambda}^2 + \rho\right] = [\text{Brans-Dicke alternative Riemann tensor } G^0_0] + 256\pi^2 S^2 \tag{34}$$

It is important to acknowledge, that the above field equations should be applied into the pseudo-General Relativistic equations, i.e., the Brans-Dicke alternative (unconventional) framework. A plausibility reasoning that substitutes an otherwise lengthy calculation, is the following: the term with spin, as well as it is added to the other general relativistic terms in (23), should be added equally to (22), because this is the Brans-Dicke equation in a general relativistic format. This equation is written in the unconventional format [23], i.e., the alternative system of equations. We could not write so simply equation (23) if the terms in it were those of conventional Brans-Dicke theory.

It is important to stress, that $\lambda(t)$ still has to obey the conservation equation (9).

Consider now exponential inflation, like we find in Einstein's theory [35]:

$$R = R_0 e^{Ht} \tag{35}$$

and, as usual in General Relativity inflationary models,

$$\Lambda = 3H^2 \tag{36}$$

For the time being, H is just a constant, defined by $H = \frac{\dot{R}}{R}$. We shall see, when we go back to conventional Brans-Dicke theory, that H is not the Hubble's constant.

From (35), we find $H = H_0 =$ constant.

A solution of Raychaudhuri's equation (23), would be the following:

$$\sigma = \sigma_0 e^{-\frac{\beta}{2}t}$$

$$\varpi = \varpi_0 e^{-\frac{\beta}{2}t}$$

$$\begin{aligned}
&\rho = \rho_0 e^{-\beta t}\\
&p = p_0 e^{-\beta t}\\
&\phi = \phi_0 e^{-\frac{\beta}{2}\sqrt{A}e^{-\frac{\beta}{2}t}}\\
&\Lambda = \Lambda_0 = \text{constant}\\
&\lambda = -\frac{\lambda_0}{2H}e^{-2Ht}\\
&n = fH\lambda_0 e^{-2Ht}\\
&S_U = SR^3 = s_0 R_0^3 e^{Ht}
\end{aligned}\tag{37}$$

In the above, $\lambda_0, \sigma_0, \phi_0, p_0, \rho_0, \beta, s_0$ and R_0, are constants, and, S_U stands for the total spin of the Universe, whose spin density equals,

$$S = s_0 e^{-\frac{\beta}{2}t} = s_0 e^{-2Ht} \tag{38}$$

while,

$$\beta = 4H \tag{39}$$

The ultimate justification for this solution is that one finds a good solution in the conventional units theory, and that the Universe must expand.

When we return to conventional units, we retrieve the following corresponding solution:

$$\begin{aligned}
&\bar{R} = R_0 \phi^{\frac{1}{2}} e^{Ht}\\
&\bar{\rho} = \rho_0 \phi^{-2} e^{-\beta t}\\
&\bar{p} = p_0 \phi^{-2} e^{-\beta t} = \left[\frac{p_0}{\rho_0}\right]\bar{\rho}\\
&\bar{\sigma} = \sigma\phi^{-\frac{1}{2}}\\
&\bar{\varpi} = \varpi\phi^{-\frac{1}{2}}\\
&\bar{\Lambda} = \Lambda_0 \phi^{-1}\\
&\bar{\phi} = \phi = \phi_0 e^{-\frac{\beta}{2}\sqrt{A}\, e^{-\frac{\beta}{2}t}}\\
&\dot{\bar{\lambda}} = \phi^{-1}\dot{\lambda}\\
&\bar{n} = n\phi^{-2}
\end{aligned}\tag{40}$$

We also have,

$$\bar{S}_U = S_U = s_0 R_0^3 e^{Ht}, \quad \text{in } c = 1 \text{ units} \tag{41}$$

As we promised to the reader, H is not the Hubble's constant. Instead, we find:

$$\bar{\Lambda} = \Lambda_0 \phi_0^{-1} e^{\frac{\beta}{2}\sqrt{A}e^{-\frac{\beta}{2}t}} \tag{42}$$

$$\bar{\rho} = \rho_0 \phi_0^{-2} e^{\beta[\sqrt{A}e^{-\frac{\beta}{2}t} - t]} \tag{43}$$

$$\bar{p} = p_0 \phi_0^{-2} e^{\beta[\sqrt{A} e^{-\frac{\beta}{2}t} - t]} \tag{44}$$

$$\bar{R} = R_0 \phi_0^{-\frac{1}{2}} e^{[Ht - \frac{1}{4}\beta\sqrt{A} e^{-\frac{\beta}{2}t}]} \tag{45}$$

$$\bar{\sigma} = \sigma_0 \phi_0^{-\frac{1}{2}} e^{-\frac{1}{2}\beta[t - \frac{1}{2}\sqrt{A}\, e^{-\frac{\beta}{2}t}]} \tag{46}$$

$$\bar{\varpi} = \varpi_0 \phi_0^{-\frac{1}{2}} e^{-\frac{1}{2}\beta[t - \frac{1}{2}\sqrt{A} e^{-\frac{\beta}{2}t}]} \tag{47}$$

$$\dot{\bar{\lambda}} = \dot{\lambda} \phi_0^{-1} e^{\frac{\beta}{2}[\sqrt{A} e^{-\frac{\beta}{2}t}]} \tag{48}$$

and,

$$\bar{H} = H \phi_0^{-\frac{1}{2}} e^{\frac{1}{4}\beta\sqrt{A} e^{-\frac{\beta}{2}t}} > 0 \tag{49}$$

From relation (48), we may calculate, by integration, the value for $\lambda(t)$:

$$\bar{\lambda}(t) = \phi_0^{-1} \lambda_0 \int e^{2H[\sqrt{A} e^{-2Ht} - t]} dt$$

Altogether, we find,

$$\bar{n} = f \lambda_0 \phi_0^{-2} H e^{2H[2\sqrt{A} e^{-2Ht} - t]}$$

The fluid obeys a perfect gas equation of state. It represents a radiation phase, if we impose,

$$p_0 = \frac{1}{3} \rho_0 \tag{50}$$

Returning to Raychaudhuri's equation, we have the following condition to be obeyed by the constants:

$$\sigma_0^2 - \varpi_0^2 = -2\pi G \left[\rho_0 + 3p_0 + 4\rho_{\lambda 0} + \kappa f \lambda_0^2\right] + 64\pi^2 s_0^2 \tag{51}$$

5 Analysis and Comments of the Results

We now investigate the limit when $t \longrightarrow \infty$ of the above formulae, having in mind that, by checking that limit, we will know which ones increase or decrease with time; of course, we can not stand with an inflationary period unless it takes only an extremely small period of time. Remember that $\beta = 4H > 0$.

We find:

$$\lim_{t \longrightarrow \infty} \bar{H} = H \phi_0^{-1/2}$$

$$\lim_{t \longrightarrow \infty} \bar{R} = \infty$$

$$\lim_{t \longrightarrow \infty} \bar{\sigma} = \lim_{t \longrightarrow \infty} \bar{\varpi} = 0$$

$$\lim_{t \longrightarrow \infty} \bar{\rho} = \lim_{t \longrightarrow \infty} \bar{p} = 0$$

$$\lim_{t \longrightarrow \infty} \bar{\Lambda} = \Lambda_0 \phi_0^{-1}$$

$$\lim_{t \longrightarrow \infty} \bar{\phi} = \phi_0$$

$$\lim_{t \longrightarrow \infty} \bar{S}_U = \infty$$

$$\lim_{t \longrightarrow \infty} \bar{n} = 0$$

By comparing the above limits, with the limit $t \to 0$, as we can check, the scale factor, total spin, and the scalar field, are time-increasing, while all other elements of the model, namely, vorticity, shear, Hubble's parameter, energy density, cosmic pressure, the number of particles injected per unit proper volume and proper time, and the cosmological term, as described by the above relations, decay with time. This being the case, shear and vorticity are decaying, so that, after inflation, we retrieve a nearly perfect fluid: inflation has the peculiarity of removing shear, and vorticity, but not spin, from the model. It has to be remarked, that pressure and energy density obey a perfect gas equation of state. The graceful exit from the inflationary period towards the early Universe radiation phase, is attained with condition (50). We have found a solution that is entirely compatible with the Brans-Dicke counterpart [9]. The total spin of the Universe grows, but the angular velocity does not [10]. By the end of inflation, the number $\bar{n}$ of injected particles has practically died away, so that, for present day Universe, the Pryce-Hoyle tensor has negligible effect: we have a kind of cosmological result. It is a remarkable novel result, that the input of matter injection, shear and vorticity, do not place any footprint into the final state of the Universe, in the aftermath of inflation ("no-hair").

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